Applied Game Theory And Strategic Behavior Chapter 1 and Chapter 2 review

Author: Siim Adamson
TTÜ 2010

Introduction

- The book „Applied Game Theory And Strategic Behavior“ is written by Ilhan Kubilay Geēkil and Patrick L. Anderson.
- Next slaids are a short review about Chapter 1 and Cahapter 2 of book „Applied Game Theory And Strategic Behavior“.

Preface

You need to learn the rules of the game. And then you have to play better than anyone else.

Albert Einstein

Purpose of this Book

The purpose of this book is to demonstrate the use of game theory techniques to address practical issues in business, applied economics, and public policy; as well as to demonstrate the benefits of strategic thinking that incorporates uncertainty about the behavior of other parties.

Organization of Book

- We introduce a brief history of game theory in Chapter 1. Readers will find the development of the field and introduction of key game theory concepts and inventions.
- If a reader does not have any interest in the history of game theory, we recommend jumping to Chapter 2, where we introduce game theory concepts and strategy. In that chapter, we show how to illustrate a game, introduce the rules of the game, key concepts, as well as strategy and strategic behavior in game theory.

Chapter 1 A Brief History of Game Theory

Why Study Games?

The field known as “game theory” was invented in the last century by mathematicians and economists as a tool to analyze both economic competition and political conflicts.

The fundamental insight of game theory was to apply the logic of games — in which players compete against each other using strategy, tactics, and effort — to events in real life.

Rapid Discoveries in the Twentieth Century

The theorists began studying game theory about a half century ago.

Key Conceptual Developments in Early Years

- In 1928, The *extensive (or tree) form* of a game, introduced by John Von Neumann.
- The definitions of a strategy, strategic behavior, and strategic actions.
- The normal form (or matrix form) of a game.
- The formulation of a game into a matrix structure.
- The concept of mixed or randomized strategy.
- The concept of individual rationality.
- Different types of information concepts, such as perfect information.
- The mini-max theorem.

Pioneers of Game Theory and Advancement

- In 1944, John Von Neumann and Oskar Morgenstern published the *Theory of Games and Economic Behavior*.
- During the 1950s, John Nash theory tools and concepts for general *non-cooperative theory* and *cooperative bargaining theory*.
- In 1950, *Strategy in Poker, Business and War* by John McDonald was published.
- In 1951 “Nash equilibrium” of a strategic game in.
- The first textbook on game theory, *Introduction to the Theory of Games,* was published in 1952 by John Charles C. McKinsey.
- In 1952 the University of Michigan along with the Ford Foundation sponsored “Design of Experiments in Decision Processes.”
- Savage principle in 1954
- Lloyd Shapley brought *conditional games* to game theory and defined the specifics of the conditional games.
- D.B. Gillies and John Milnor developed the first *continuous* game theoretical models.
- Harold Kuhn contributed to the field with his work on behavior strategies.
- Melvin Dresher and Merrill Flood of the RAND Corporation developed the structure of the prisoner’s dilemma in Santa Monica in the 1950s.

- In 1955, it was applied to philosophy by British philosopher R. B. Braithwaite in his book *Theory of Games as a Tool for the Moral Philosopher.*
- In the late 1950s Kuhn, Kissinger, and Schelling contributed to the field while developing cold war strategies.
- In the In late 1950s is the use of the *Folk Theorem* to show the solid relationship between repeated and cooperative games.
- In 1957, *Games and Decisions,* by R. Luce and H. Raiffa, was published.
- John C. Harsanyi developed games with incomplete and asymmetric information.
- In 1960, *The Strategy of Conflict* by Thomas Schelling was published. Schelling introduced the *focal point* concept, also known as the Schelling point.
- In 1966 Harsanyi introduced cooperative games, in which commitments, contracts, agreements, threats, and promises are enforceable.
- In 1969, D.K. Lewis formalized the *common knowledge* assumption.
- In 1972, the *International Journal of Game Theory* was first published. It was founded by Oskar Morgenstern.
- In 1973, Harsanyi introduced the idea of explicit randomization in game theory.
- In 1974, Robert Aumann introduced the *correlated equilibrium*.
- In the 1970s the game theory was applied to biology by John Maynard Smith in his work on evolutionary stable strategy (ESS).

Game Theory's Evolution during the Last Three Decades

- In 1981, Elon Kohlberg published Some Problems with the Concept of Perfect Equilibria.
- In 1982, "Sequential Equilibria" by David Kreps and Robert Wilson was published in *Econometrica*.
- In 1984, David Pearce and Douglas Berheim independently introduced the idea of rationalizability in their papers *Rationalizable Strategic Behavior* and *the Problem of Perfection*.
- In 1984, *The Evolution of Cooperation* by Robert Axelrod was published.
- In 1986, Elon Kohlberg and Jean-Francois Merterns published *On the Strategy Stability of Equilibria.*
- In 1988, *A General Theory of Equilibrium Selection in Games* by John Harsanyi and Reinhard Selten was published.
- In 1989,the journal *Games and Economic Behavior* (GEB) was first published.
- In 1990, *A Course in Microeconomic Theory* by David Kreps was published.
- In 1994, *Game Theory and the Law* by Douglas Baird, Robert Gertner and Randal Picker was published.

Chapter 2 Strategy and Game Theory Concepts

Thus, what is of supreme importance in war is to attack the enemy's strategy.

Sun Tzu

Game theory

- interactive decision-making environment
- offers valuable tools for solving strategy problems

Game Theory, Strategy, and Strategic Behavior

- A **game-theoretic model** is an environment where each decision-maker's actions interact with those of others.
- Behavior that involves such interactive decision-making is called ***strategic.***
- The set of actions and moves by each player is called *strategy.*
- We will consider **"strategic" behavior** as behavior serving the self-interest of the person

More on Strategic Behavior and Strategy

- We will consider “strategic” behavior as behavior serving the self-interest of the person, based on the person’s own subjective evaluation of likely events and the possible actions of other players; with the potential rewards and risks being considered over single or multiple period(s).

Game Theory and Strategic Behavior in Business

- Strategic behavior occurs regularly among executives, managers and investors in business.
- In decision-making situations, the person confronts not only uncertainty about future states of nature, but also uncertainty about actions that other persons will take.
- Using any of the definitions of "strategy" introduced above, this behavior is called "strategic behavior."

Consumer Behavior, Utility Theory, and Game Theory

- The important assumptions of game theory is that economic agents are rational players.
- The goal is to maximize well-being, i.e., utility.
- To model consumers' preferences, we use utility functions.
- Economists say a bundle ($x1$, $x2$) is preferred to a bundle ($y1$, $y2$) if and only if the utility of ($x1$, $x2$) is larger than the utility of ($y1$, $y2$). Symbolically, ($x1$, $x2$) is preferred to ($y1$, $y2$) if and only if $u(x1, x2) > u(y1, y2)$. This assumption is very important; it is called **an axiom of utility theory**. If consumer prefers X to Y and Y to Z then this can be marked as $U(X) > U(Y) > U(Z)$.

Cardinal Utility

- Utility theories where the magnitude of utility is important are called “cardinal utility theories.”
- Cardinal utility theory asserts that the level, as well as the order, of utility gained from a bundle of goods and services is significant.
- For example, one prefers a specific bundle at least three times more than another if he is willing to pay three times as much for that bundle.

Choice Behavior and Game Theory

- ***Choice behavior*** is determining whether one bundle or another will be preferred.
- In our game theory models, we consider which strategy brings greater utility.

Utility Functions and Game-Theoretic Models

- Some game-theoretic models require a ***utility function***.
- A **utility function** maps the utility of a bundle of goods and services to a real number.
- Examples of utility functions:
  - multiplicative function $u(x1, x2) = x1x2$,
  - additive function $u(x1, x2) = ax1 + bx2$,
  - maximum function $u(x1, x2) = \max\{ax1, bx2\}$.

Utility Theory and Payoffs

- In the real world, properly constructing the payoff matrix is a critical, and often difficult, step.
- This difficulty arises from the fact that a person's utility is rarely defined by any onedimensional measure, such as a price, quantity, or size, of a cash payment.

Game-Theoretic Models and Illustration

- Game theory models describe strategic interaction among many players (rational decisions, max. utility).
- **Sequential interaction** refers to each player taking action in a sequence of turns
  - player aware of
    - the actions taken in previous turns,
    - current action(s) will affect future actions.
- **Simultaneous interactions** occur when players take actions concurrently, in ignorance of the others' current actions

The Payoff Matrix and Tree Diagram

- To analyze sequential-move and simultaneous-move games, we the **payoff matrix** and the **tree diagram** can be used.
- The payoff matrix illustration in game theory is also called the **“normal form”** of the game.
- A **tree diagram** is tool for illustrating and analyzing games, in which players act sequentially.
- **“backward induction”** - *look ahead and reason back* (players observe the other players’ future moves and use them in assessing their best current move).
- Tree diagrams, generally used for sequential or the non-simultaneous games, are also called **“decision trees,”** or the ***extensive form* of the game**.

Strategic Thinking and Simultaneous- and Sequential-Move Games

- In **sequential-move games**, the current course of action taken by the player is based on ones expectation of what the other players' future strategy and action will be.
- In contrast, **simultaneous-move game** interaction can be more difficult for players. This is because players must guess what the other player is anticipating at the moment, and respond accordingly, as well as anticipate how these actions affect future outcomes of the game.

Rules of the Game

The nature of a game theoretical model is determined by the rules. The key rules of any game:

- ***Players.*** How many players do we have in a game? Are their interests matching or conflicting?
- ***Information.*** What information does each player possess? Do they have *complete, symmetric,* or *perfect* information regarding each other's actions and payoffs? What are the moving sequences of players?
- ***Actions or Strategies.*** What actions or strategies are the players allowed to have? What are the specifics of interaction between players? Are they allowed to communicate?
- ***Payoffs.*** What are the possible outcomes for each player? What is the utility or expected utility for each player at the end of the game for every action they are allowed to have?

- **Players**
  - Players are rational economic agents, make decisions with a well-defined set of actions and strategies.
  - Their goals are to maximize their utility or expected utility.
- **Information**
  - Information is the knowledge each player has about the game.
  - Important implicit assumption of game theory is that the structure of the game is common knowledge.
  - We have three main categories for the information structure of a game :
    - Perfect vs. imperfect information,
    - Complete vs. incomplete information,
    - Symmetric vs. asymmetric information.

- ***Perfect vs. Imperfect Information***
  - Perfect information means that no moves are simultaneous, and each player knows the sequence of moves and where players move.
  - All simultaneous-move games are games of imperfect information.
  - An incomplete or asymmetric information game is also a game of imperfect information.
- ***Complete vs. Incomplete Information***
  - In a game of incomplete information, there are some uncertainties about
    - the actions of players,
    - the moving sequence of the game,
    - the payoffs.
  - A game of incomplete information might include probabilities at some of the nodes in the game.
  - A game of incomplete information is also a game of imperfect information.

- ***Symmetric vs. Asymmetric Information***
  - In a game of symmetric information, players have the same elements in their information sets.
  - Otherwise, a game is called a “game of asymmetric information.”
  - In asymmetric information games, players have different information regarding each other’s moves or payoffs.
- **Set of Actions and Strategies**
  - Player has an action set that includes their possible moves or strategies.
  - Players determine their strategies based on the information available
- **Payoffs**
  - Payoffs are what players receive at the end of the game.
  - The nature of games is that the payoffs differ depending on the actions of the players.
  - Possible payoffs are visualized in a payoff matrix.

Strategy and Equilibrium

- A "strategy" is an order of moves determined in advance of some events by an individual player.
- **Dominant and Dominated Strategies**
  - If any player follows a *dominant strategy*, the player will get the best possible payoff regardless of what the other player(s) will do.
  - A **dominant strategy** is the ***optimal*** strategy for a player no matter what the other player(s) does (do).
- ***Dominated Strategies***
  - A **dominated strategy** is a strategy that is **worse** than another strategy available for the player.
- **Rational players** be expected to **play** their **dominant strategies** (if they have any) and avoid their dominated strategies.

- ***Equilibrium***
  - An ***equilibrium*** in game theory is defined as a **stable outcome**, based on the payoffs received by players at the end of the game.
  - **an equilibrium point** – players have no incentive to deviate from that point.
  - **equilibrium point** in a game = **the *solution*** of the game.
- ***Dominant Strategy Equilibrium***
  - In parlor games, there is no strategy that is dominant.
  - Players either
    - have dominant strategies,
    - learn their best strategies over time and begin to play them repeatedly.
  - every player in a game has a dominant strategy.
  - no player has a dominant strategy.
  - If there is a **dominant strategy** for **each player**, then we have a ***dominant strategy equilibrium*** for that game.
  - If there is a **dominant strategy for only one player**, we have a **dominant strategy equilibria** in a 2-player game.
  - If it is an **n-player game**, we **may or may not** have a **dominant strategy equilibrium**.

- ***Nash Equilibrium***
  - We do not have dominant strategy equilibria in all games.
  - In a two-player simultaneous-move game, we call a pair of strategies a *Nash equilibrium*, if player I's choice is optimal based on player II's choice, and player II's choice is optimal based on player I's choice.
  - If a game is a non-simultaneous (sequential) game, the first mover has the advantage and is able to dictate an equilibrium.
  - Do Nash equilibria exist in every game? The answer is no.

- ***Note on Dominant Strategy Equilibrium and Nash Equilibrium***
  - The Nash equilibrium occurs in a broader spectrum of games.
  - A game has a Nash equilibrium if there exists a set of strategies such that each player optimizes his utility given the other players' actions.
  - A Nash equilibrium is quite stable, because no player has an incentive to deviate from his *Nash strategies*.
- ***Subgame Perfect Nash Equilibrium***
  - ***Subgame*** is a **smaller portion of a game** starting at a specific node of an entire game.
  - We call an equilibrium a ***subgame perfect Nash equilibrium***, if **every subgame of the entire game has a Nash equilibrium** based on players' strategies.

- ***Mixed Strategies; Repeated Games***
  - **“pure strategies”** - players making one choice and playing the game with that choice only.
  - If games are played more than once, we call them ***repeated* games.**
  - If player change strategy after evry round, this is called a ***mixed strategy.***
- ***Maximin Strategy***
  - The maximin strategy pertains to a two-person zero-sum game.
  - If a player (player I) attempts to take action(s) to reduce the other player’s (player II) payoff, player II will take the action(s) that will give him the maximum minimum payoff.
  - Most games are not *zero-sum games*, the maximin strategy is often not applicable.

- **Sequential Games and Problem Solving**
  - Rules and information are very important in game theory, as **changing** the **rules** will often **change** the **outcome** of the game.

Complex Games and Games by Categories

- In real life, games are more complex.
- Simple games to illustrate game theoretic concepts were used.
- **n-Person Games**
  - We call games with more than two players, n-person games where n can be 3, 4 or more.
  - In n-person games, *power* is the key element for the outcome of the game.
  - We define power here as the ability to affect the outcome of a situation to one's favor.

Different Categories of Games

- ***Zero-Sum Games vs. Non-Zero-Sum Games***
  - In zero-sum games, one player's gain is another player's loss.
  - In zero-sum games, the sum of the payoffs of all players should be zero.
  - Zero-sum games are also called *constant-sum games*.
  - Most of the games are not zero-sum games.
  - In non-zero-sum games, all players could win or lose together.
  - In zero-sum games, players have no common interests.
  - In non-zero-sum games, players have common and conflicting interests.
  - In non-zero-sum games, players have win-win situations.

- ***Static vs. Dynamic Games; Repeated Games***
  - In real life, most of the games are played more than once.
  - Dynamic games - play may unfold differently with a sequential game; that is where the players play the game more than once consecutively.
  - Repeated games are dynamic.
  - In dynamic games, unlike static games, players observe other players' behaviors, modify their strategies accordingly, and develop reputations about their own behavior.

- ***Cooperative vs. Non-Cooperative Games***
  - A game can be cooperative or non-cooperative by nature.
  - A game in which players are allowed to cooperate with each other on a joint strategy is called a "cooperative game."
  - In cooperative games, agreements, commitments and threats are binding and enforceable.
  - A game in which players are not allowed to cooperate or negotiate on a contract is called a "non-cooperative game", commitments are not enforceable.

Other Key Game Theory Concepts

- **Threats and Rewards (Promises)**
  - In game theory, players can achieve a strategic advantage through the *response rule*.
  - A response rule sets one's action(s) as a response to another's action(s).
  - The response rule can be defined in two ways: threats and rewards.
  - Threats and promises are essentially the same; both are messages that players can send to each other to affect the other player in choosing a certain action.
  - With a threat, failure to cooperate results in some type of negative payoff.
  - With a reward or promise, cooperation results in some type of positive payoff.
  - Both threats and rewards can be defined as compellent or deterrent.
  - A compellent threat is meant to induce action from another, while a deterrent threat is meant to prevent future action from another.

- ***Credibility***
  - The *credibility* - If the threat or promise looks fundamentally unrealistic, then the threat or promise is not credible.
- ***The Threat as a Strategy***
  - Threats and rewards are strategic moves.
  - Threats and rewards must be credible to influence the behavior of others.
  - Smart players often display a pattern of fulfilling threats and promises throughout the game.
  - When a player needs to offer a reward, he should not promise more than necessary to influence behavior.
  - Threats and promises of disproportionate scale can undermine the reputation of a player.

- **Games of Chance: Uncertainty and Risk**
  - In game theory, chance and uncertainty are very important concepts.
  - In games of chance nature determines if one player is a winner or loser, and how much he wins.
  - Games of chance might be oneplayer games.
  - Games of chance involve either risk or uncertainty, or both.
  - **Backgammon** – games involve both chance and strategy.
  - Even in pure chance games, **by randomizing** a player **can develop a strategy.**
  - Uncertainty is brought by nature and/or other players.
  - One-player games against nature are not the subject of this book.
  - Goal is to analyze strategic behavior and strategy, interactive decision-making environments, which involve two or more players.

Examples

- http://publik.delfi.ee/games/?genre=puzzle&game=tripstrapstrull

- Questions ?